Ch9 Canvas Quiz Key

1. Suppose that a market research firm is hired to estimate the percent of adults living in a large city who have cell phones. Five hundred randomly selected adult residents in this city are surveyed to determine whether they have cell phones. Of the 500 people surveyed, 421 responded yes - they own cell phones. Using a 95% confidence level, compute a confidence interval estimate for the actual proportion of adult residents of this city who have cell phones.

   (a) Use the given information to estimate the actual proportion of adult residents of this city who have cell phones.

   (a) 84.2%

   (b) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

   (c) Compute/find the value of the margin of error.

   (c) 3.2%

   (d) Interpret the meaning of the margin of error in the context of this problem.

   (e) Construct a 95% confidence interval for the actual proportion of adult residents of this city who have cell phones.

   (e) (81%, 87.4%)

   (f) <u>Communicate the Result</u>: Interpret the confidence interval.

   (g) <u>Communicate the Result</u>: Interpret the confidence *level.*

1a.

$n = 500$

$x = 421$

$\hat{p} = \frac{x}{n} = \frac{421}{500} = 0.842$ or 84.2%

1b. The sample is random. There are 421 successes (yes responses) and 500 - 421, or 79 failures (no responses) in the sample. So, there are at least ten successes and at least ten failures in the sample, Therefore we conclude that the sampling distn. of $\hat{p}$ is approximately normal.

1c. $M = 1.96 \cdot \sqrt{\frac{p(1-p)}{n}} \approx 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= 1.96 \cdot \sqrt{\frac{0.842(1-0.842)}{500}}$

≈ 0.032 or 3.2%

(1d.) It is likely that our estimate for the actual percent of adults in the city who have cell phones (84.2%) differs from the actual percent by not more than 3.2%.

It would be unusual for our estimate (84.2%) to differ from the actual proportion of adults in the city who have cell phones by more than 3.2%.

For 95% of random samples, the estimation error will be less than the margin of error. That is,

$$|\hat{p}-p| < M \text{ for } 95\% \text{ of } \hat{p}.$$

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It would be sufficient to just write this.

(1e.) $\hat{p} - M = 84.2\% - 3.2\% = \boxed{81\%}$

Also, $\hat{p} + M = 84.2\% + 3.2\% = \boxed{87.4\%}$

So the 95% confidence interval estimate for p is (81%, 87.4%).

(1f) The marketing research firm can be 95% confident that the actual proportion of adults living in the large city who have cell phones is somewhere between 81% and 87.4%.

(1g) The method used to construct this interval estimate is successful in capturing the actual value of the population proportion.

2. https://www.democratandchronicle.com/story/news/politics/albany/2020/04/20/coronavirus-new-york-begins-antibody-testing-what-means/5164144002/

April 24, 2020 — New York's first random sample of coronavirus antibodies shows that 13.9% of those tested in the state had coronavirus antibodies in their system, meaning they have contracted and recovered from the virus, New York Governor Andrew Cuomo said Thursday. That suggests that 2.7 million people have been infected statewide.

The survey was taken from a sample size of about 3,000 people found outside their homes, shopping at essential businesses, such as grocery stores, which remain open. Results show antibodies in 12% of women and 15.9% of men, but a disproportionate rate of antibodies in black and Latino New Yorkers. Cuomo said the spread likely reflects the regional breakdown of the state.

The rate was even higher in New York City, with about 21% showing antibodies, and on Long Island, with 16.7%. According to Cuomo, if the results are proven accurate and the overall infection rate in New York is about 13.9%, the death rate from coronavirus may be lower than some estimates. In this problem, we are interested in estimating the actual proportion of New Yorkers who have coronavirus antibodies in their system.

(a) About how many people in the random sample tested positive for coronavirus antibodies?

(a) 417

(b) About how many people in the random sample tested negative for coronavirus antibodies?

(b) 2583

(c) Verify that the conditions needed in order for the margin of error formula to be appropriate are met.

(d) Compute/find the value of the margin of error. (Use a 99% confidence level)

(d) ____________

(2a) 13.9% of 3,000 people

$= 0.139 \times 3{,}000 = 417$

(2b)

$$\begin{array}{r} 3000 \\ -\ 417 \\ \hline 2583 \end{array}$$

(2c) The sample was a <u>random sample</u> of New Yorkers found outside their homes. There are at least 10 successes and failures in the sample, since 417 people had had the virus (success), and 2,583 people had not (failure). So, the sampling distr. of $\hat{p}$ is approx. normal.

(2d)

$$M = 2.58\sqrt{\frac{p(1-p)}{n}} \approx 2.58\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 2.58\sqrt{\frac{0.139(1-0.139)}{3000}}$$

$$\doteq 0.016 \text{ or } 1.6\%$$

(e) Interpret the meaning of the margin of error in the context of this problem.

(f) Construct a 99% confidence interval for the actual percentage of New Yorker who have coronavirus antibodies in their system

(f) ________

(g) Communicate the Result: Interpret the confidence interval.

(h) Communicate the Result: Interpret the confidence *level*.

(i) According to your results, about how many New Yorkers are predicted to have already contracted and recovered from Covid-19.
Was Governor Cuomo correct in his estimate?

(i) ________

3. Georgia, the home of the CDC headquarters, wants to conduct a random sample of Georgians to estimate the actual percentage of Georgians who have already have already contracted and recovered from Covid-19. What sample size would you use in order to estimate this proportion with a margin of error of 0.01?

$$n = p(1-p)\left(\frac{1.96}{m}\right)^2 = (0.5)(1-0.5)\left(\frac{1.96}{0.01}\right)^2$$

$$= (0.5)(0.5)(196)^2 = \boxed{9{,}604}$$

(2e) Assuming that the sample of 3,000 people represents all new yorkers, and was selected in a reasonable way, it would be unusual for the estimate (13.9%) to differ from the actual proportion of New Yorkers who had had the virus by more than 1.6%. For 99% of all random samples, the estimation error will be less than 1.6%

(2f) $\hat{p} - m = 13.9\% - 1.6\% = \boxed{12.3\%}$

and $\hat{p} + m = 13.9\% + 1.6\% = \boxed{15.5\%}$

(2g) Assuming the sample was truly random, and representative of all New Yorkers, we can be 99% confident that the actual percentage of New Yorkers who have already contracted and recovered from covid-19 is between 12.3% and 15.5%.

(2h.) The method used to construct this interval estimate is successful in capturing the actual percent of New Yorkers who have coronavirus antibodies about 99% of the time.

(2i) A google search states that the population of New York is approximately 19.45 million. This means it is likely that between 2.4 million and 3.0 million New Yorkers have already contracted and recovered from the virus.

12.3% of 19.45 million people

$= 0.123 \times 19.45 \approx \boxed{2.39 \text{ million}}$

15.5% of 19.45 million people

$= 0.155 \times 19.45 = \boxed{3.01 \text{ million}}$

3

$$n = p(1-p)\left(\frac{1.96}{M}\right)^2$$

$$= 0.5(1-0.5)\left(\frac{1.96}{0.01}\right)^2$$

$$=(0.5)(0.5)(196)^2$$

$$= 9{,}604$$

<u>no</u> value of p is given, so we use $p=0.5$.

Also,

$M=0.01$ or 1%

Georgia would need a random sample as big as 9,604 to estimate the actual percentage of Georgians who had contracted and recovered from Covid-19 to within 1%.